Generalized Solutions of Piezoelectric Vibration-Based Energy Harvesting Structures Using an Electromechanical Transfer Matrix Method

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Piezoelectric vibration-based energy harvesting (pVEH) offers much potential as renewable energy structures. Within the literature, often geometry-specific models are developed, making designs of new structures difficult. In this work, a generalized linear algebraic method is developed. The method incorporates the transfer matrix method (TMM) into the well-accepted distributed parameter electromechanical model for a composite-piezoelectric, Euler–Bernoulli beam. The result is an electromechanical TMM which is highly accurate at predicting both structural and energy harvesting performances for a wide variety of designs which have chainlike topologies. A simplification is made within the method to model structures which operate solely within bending modes, reducing the computation to analyses of only four-by-four state transition matrices, regardless of structural complexity. As many applications aim to optimize the large bending mode piezoelectric effect, this simplification does not limit the versatility of the method. To demonstrate the validity of this statement, comparisons were performed to evaluate the accuracy of the method's predictions for six piezoelectric topologies, including a unimorph without a tip mass, a bimorph with a tip mass, several partial-length bimorphs without a tip mass, and three different multibeam bimorph structures with inline and folded-back designs. The results show differences no greater than 2.24% for the first and second natural frequencies of the structures. Likewise, the method yields excellent predictions for the mode shapes, their slopes, and the voltage frequency responses, especially within the ±10% bounds of the natural frequencies. Thus, the future design of new structures is shown to be simplified using this generalizable method.

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1 Introduction

The last decade has seen a surge in the literature concerning renewable energy sources, specifically for remote, low-power applications. The field known as energy harvesting focuses on developing devices that convert available energy from the ambient environment into useable electrical form [1–3]. The appeal of energy harvesting is greatest for long-term applications where it offers a potential financial advantage over wired and battery operated systems by reducing or eliminating maintenance costs. Although a variety of ambient energy sources are available, VEH has received a significant amount of attention due to its ability to be implemented on man-made structures, where vibration is omnipresent [1]. A common transduction type for this renewable energy source has been piezoelectrics, which convert mechanical deformation into electrical energy. This energy transfer is described by constitutive relations known as the direct piezoelectric effect.

For pVEH, the amount of energy harvested is determined by mechanically exciting the composite structure with the ambient vibration and measuring the resultant voltage output across a resistive load, or stored into a capacitive device [2]. When designing pVEH structures, two analytical modeling methods are common: (1) lumped parameter models, typically single degree-of-freedom [4,5] and (2) distributed parameter models, or multiple degrees-of-freedom [6–11]. Lumped parameter methods provide simple and effective models when vibrating near a single resonant frequency, but are difficult to design with since their coefficients are typically determined experimentally. Distributed parameter models are more accurate, however, the caveat is that these models are typically much more complex and are often specific to the design geometry of each structure.

In this paper, a generalized, linear algebraic method is developed which broadens the versatility of distributed parameter models beyond geometry specific solutions, allowing for the design of any pVEH structure with chainlike topologies. Such designs include those most commonly used in pVEH applications, such as single structures with full and partial length unimorphs, full and partial length bimorphs, both with and without tip masses, and multibeam designs just to name a few. To explain briefly, the generalized method is developed using a combination of the TMM [12] and an existing distributed-parameter pVEH model for prismatic, Euler–Bernoulli beam structural members [7]. Two major conditions are imposed for its proper usage: (1) the pVEH structure must have a chainlike topology which can be geometrically decomposed into prismatic beam elements with discontinuities, which may include lumped masses, at their junctions, and (2) Euler–Bernoulli beam theory must be applicable for the prismatic beam elements. While similar TMM approaches have been previously used in the pVEH literature [13,14], this work separates itself in two ways: (1) it develops a simplified analysis for structures which operate solely in bending modes, and (2) it provides the first experimental validation of this method for multiple structure designs, including all of the following analyses: natural

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